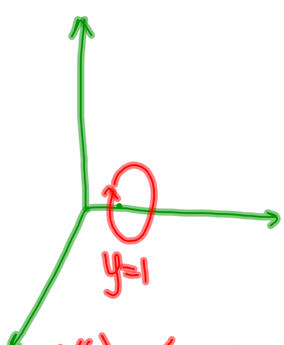
Arc length

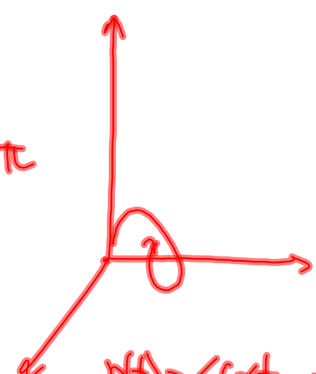
$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_c^d \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \longrightarrow L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$\sum \sqrt{\frac{(\Delta x)^2}{(\Delta t)^2} + \frac{(\Delta y)^2}{(\Delta t)^2}}\, \Delta t$$



$0 \le t \le 2\pi$



$r(t) = \langle \cos t, 1, \sin t \rangle$

$$L = \int_0^{2\pi} \sqrt{(-\sin t)^2 + 0^2 + (\cos t)^2}\, dt$$

$$= \int_0^{2\pi} 1\, dt = 2\pi$$

$r(t) = \langle \cos t, t, \sin t \rangle$

$$L = \int_0^{2\pi} \sqrt{(-\sin t)^2 + 1^2 + (\cos t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{2}\, dt = 2\pi\sqrt{2}$$

$r(t) = \langle t, 2t, t^2 \rangle$

Find the length of the curve for $0 \le t \le 1$

$$L = \int_0^1 \sqrt{1^2 + 2^2 + 4t^2}\, dt = \int_0^1 \sqrt{5 + 4t^2}\, dt$$

$\sin\theta = \frac{\sqrt{5}}{\sqrt{5+4t^2}}$

$\sqrt{5+4t^2} = \frac{\sqrt{5}}{\sin\theta}$

$\tan\theta = \frac{\sqrt{5}}{2t}$

$t = \frac{\sqrt{5}}{2}\cot\theta$

$dt = \frac{\sqrt{5}}{2}(-\csc^2\theta)\, d\theta$

$$= \int \frac{\sqrt{5}}{\sin\theta}\left(-\frac{\sqrt{5}}{2}\csc^2\theta\right) d\theta$$

$$= -\frac{5}{2}\int \csc^3\theta\, d\theta$$

$$= -\frac{5}{2}\int \csc\theta(1 + \cot^2\theta)\, d\theta$$

$$= -\frac{5}{2}\left(\int \csc\theta\, d\theta + \boxed{\int \csc\theta \cot^2\theta\, d\theta}\right)$$

$$= -\frac{5}{2}\left(-\ln|\csc\theta + \cot\theta| + -\cot\theta\csc\theta - \int \csc^3\theta\, d\theta\right)$$

$$\int \frac{\csc\theta(\csc\theta + \cot\theta)}{\csc\theta + \cot\theta}\, d\theta = -\ln|\csc\theta + \cot\theta| + C$$

$$\int \frac{\sec\theta(\sec\theta + \tan\theta)}{\sec\theta + \tan\theta}\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$-\frac{5}{2}\int \csc^3\theta\, d\theta$

$$= -\frac{5}{2}\left(-\ln|\csc\theta + \cot\theta| - \cot\theta\csc\theta - \int \csc^3\theta\, d\theta\right)$$

$$-5\int \csc^3\theta\, d\theta = -\frac{5}{2}\left(\ln|\csc\theta + \cot\theta| + \cot\theta\csc\theta\right)$$

$$\int \csc^3\theta\, d\theta = -\frac{1}{2}\left(\ln|\csc\theta + \cot\theta| + \cot\theta\csc\theta\right)$$

$$L = \frac{5}{4}\left(\ln|\csc\theta + \cot\theta| + \cot\theta\csc\theta\right)$$

$$\int \csc\theta \cot^2\theta\, d\theta \qquad \int \frac{\cos^2\theta}{\sin^3\theta}\, d\theta$$

$$= -\cot\theta\csc\theta - \int \csc^3\theta\, d\theta$$

$$\int \csc\theta \cdot \cot^2\theta\, d$$

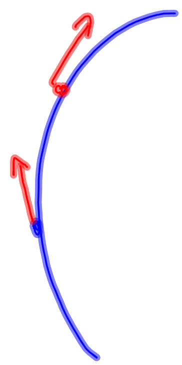
$f = \cot\theta \qquad dg = \csc\theta\cot\theta\, d\theta$

$df = -\csc^2\theta\, d\theta \qquad g = -\csc\theta$

$$= -\cot\theta\csc\theta - \int \csc^3\theta\, d\theta$$

$$K = \left|\frac{\Delta T}{\Delta s}\right| = \left|\frac{dT}{ds}\right| = \left|\frac{\frac{dT}{dt}}{\frac{ds}{dt}}\right| = \frac{\left|\frac{dT}{dt}\right|}{\left|\frac{ds}{dt}\right|}$$

curvature



$$\frac{ds}{dt} = \left|\frac{dr}{dt}\right|$$

$$= \frac{|T'|}{|r'|}$$

$$T = \frac{\frac{dr}{dt}}{\left|\frac{dr}{dt}\right|}$$

$$r(t) = \langle a\cos t, a\sin t, 1 \rangle$$

$$r'(t) = \langle -a\sin t, a\cos t, 0 \rangle$$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{\langle -a\sin t, a\cos t, 0 \rangle}{a} = \langle -\sin t, \cos t, 0 \rangle$$

$$T'(t) = \langle -\cos t, -\sin t, 0 \rangle$$

$$K = \frac{|T'(t)|}{|r'(t)|} = \frac{1}{a} = \boxed{\frac{1}{a}}$$